Working Time Reduction, Unpaid Overtime Work and Unemployment *

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Abstract

This paper examines the impact that a reduction in the number of hours in a standard work week has on unemployment under a wage-posting framework combined with a matching model. Many studies have addressed this problem, some of which have developed an extension of an equilibrium search model with a Nash bargaining approach. However, the negotiation between an employer and a worker is not the only factor in determining the contents of a labor contract. We thus focus on the case in which each employer unilaterally determines the wage level and the number of hours worked and job seekers can only accept or reject an offer. The model takes into account the amount of unpaid overtime that employees work. Consequently, a reduction in the number of hours in a standard work week can decrease unemployment, provided that overtime work (as in Japan) is not fully compensated. In addition, we also find that this policy may increase the proportion of firms that post higher wage offers when employment is not frequently terminated.

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1 Introduction

Whether or not a reduction in working time has a positive effect on employment is a matter of concern for policy-makers and economists. One of the most famous instances supporting the effectiveness of such a policy is the work sharing practiced in the Netherlands in the 1990s, which increased part-time jobs and resulted in drastically lower unemployment. This is called the "Dutch miracle" (for more details, refer to Salverda (1999) and Van Oorschot (2000)). Many theoretical and empirical studies, however, indicate that a policy of work time reduction does not always reduce unemployment. In other words, there is no consensus concerning the effectiveness of a work time reduction policy. Many studies address this problem by developing an extension of an equilibrium search model with a Nash bargaining approach. However, negotiation between an employer and a worker is not the only factor determining the contents of a labor contract. For this reason, we here combine the wage-posting framework with a matching model to examine the impact that a reduction in the number of hours in a standard work week (hereafter, standard hours) has on unemployment. It can be expected that our study will provide new guidelines for examining the employment effect of a reduction in the number of hours worked.

By introducing working time into the standard equilibrium search framework (referred to as the Diamond-Mortensen-Pissarides model), Marimon and Zilibotti (2000) and Rocheteau (2002) are able to describe economic circumstances in which a reduction in working time stimulates job creation. [1)] Marimon and Zilibotti (2000) find that only a moderate reduction in work time increases employment. Rocheteau (2002) incorporates the possibility of worker moral hazard into the model and concludes that reducing working time reduces unemployment only in countries with high unemployment. FitzRoy et al. (2001)(2002) discuss the impact of this policy on employment in a general equilibrium framework containing a government sector. They find that if the bargaining power of a labor union is weak and the power of a firm is strong, the policy has a desirable effect. Furthermore, as stated in Booth and Schiantarelli (1987), the policy has a negative effect on employment when a union has monopolistic bargaining power. Brunello (1989) empirically examines the effect of a working time reduction policy by using data from the Japanese manufacturing sector. The study suggests that employers will increase the amount of overtime that employees work and that employment will decrease as a result of the policy. [2)]

Most of the above studies assume that wages and hours worked are determined by negotiation between a worker and a firm or between a union and a firm. This assumption is applicable to cases where working conditions are negotiated based on a centralized form, as is the case in some European countries, or when an individual worker and an employer negotiate work conditions, as is the case in the U.S. However, this would not be an ap-

1) Contensou and Vranceanu (2000) explore the theoretical, empirical and political implications of several problems concerning working time. They develop a study of work time based on a matching model and conclude that a policy of work time reduction may expand employment.

2) Since the 1980s, Japanese employment practices have been criticized by other developed countries for the remarkably long hours worked, and as a consequence, the number of hours considered to be standard has been gradually reduced to 40 hours. According to the time series data on unemployment provided by the Statistics Bureau in the Ministry of Internal Affairs and Communications, the Japanese economy experienced an exceptionally high unemployment rate between 2001 and 2003. These events have led to a discussion of the introduction of a work-sharing system suitable for the Japanese economy.

propriate approach for an environment in which the labor force is not particularly mobile or employees enter jobs immediately after graduating from a university, as is the case in Japan. Since recent graduates generally have few skills and little experience, it is evident that they are not mobile and do not have much bargaining power. Furthermore, Michelacci and Suarez (2006) state that the wage-posting system is suited to describing the Japanese economy because some examples from literatures, such as Klein (1992) and Baron and Kreps (1999), suggest that predetermined wage structures are prevalent in Japan. Because of these foundations, we examined how a reduction in the standard hours (one means of work-sharing) affects unemployment when workers have nothing to do with the determination of wage payments and the number of hours worked and can only decide whether to accept or reject an offer. In practice, there are a considerable number of people under these circumstances, and the model developed here will focus on the labor market from a point of view different from the standard search-matching framework (e.g., that developed by Pissarides (2000)). We describe these situations by using the wage-posting framework developed in Burdett and Mortensen (1998) and Quercioli (2005).

Burdett and Mortensen (1998) in particular represent pioneering work in this field. In a wage-posting model, firms post their wage offer (or a utility offer as in Hwang et al. (1998)) before a search process begins. If wage offers are greater than the benchmark wage of job seekers, a worker-firm pair is formed. The success rate for each employer of hiring a worker depends on the wage level and on several market conditions, such as the job-offer arrival rate. Since higher (or lower) wages attract more (or fewer) applicants with lower (or higher) flow profits per worker, this trade-off generates many alternatives for maximizing expected profits. Thus, wage dispersion is the equilibrium outcome, and furthermore, a continuous distribution arises from analytical characterization in this framework. The standard Burdett-Mortensen models, by contrast, suppose that the offer arrival rate for workers is given as exogenous, and they neglect the job flow (the job offer arrival rate is exogenously given). This does not seem realistic, so we therefore introduce a job creation mechanism described in the model, combined with a matching framework, as in Mortensen and Pissarides (1999), Rosholm (2000) and Albrecht and Vroman (2005). Using this model, we examine the impact that a reduction in the standard hours has on job creation (moves which entail creating new job vacancies in this paper and not the number of newly filled jobs).

The model used in this paper also incorporates a determination of the number of hours worked by employers and the existence of unpaid overtime. Many studies, including Trejo (1991), Hunt and Katz (1998), Bell and Hart (1999*a*)(1999*b*), Contensou and Vranceanu (2000), Marimon and Zilibotti (2000), Hart (2004) and Mizunoya (2005), deal with overtime work. However, there is little research that theoretically examines the impact of work-time regulation on employment when overtime work is taken into consideration. [3] Furthermore, the study performed by Mizunoya (2005) is almost unique in that it provides an

3) Trejo (1991) investigates the economic consequences of overtime pay regulation. He explores the hypothesis that changes in the statutory overtime premium will be neutralized by canceling out adjustments in the straight-time hourly wage rate. However, the data analyzed in that paper suggest that these adjustments are not so large that the effect of overtime pay regulation is not offset completely. Thus, this regulation may expand employment, and we demonstrate in this paper that such overtime regulation leads to similar desirable results.

international comparison of the amount of unpaid overtime work. According to Mizunoya (2005), unpaid overtime is a prevailing phenomenon in many developed countries; this will be discussed further in Section 2. Thus, the effect of the level of payment for overtime work on the impact of a policy of working time reduction is an important topic, and the validity of this policy should be judged according to such theoretical implications. In this paper, we suppose that employers provide insufficient payments for overtime work in that the hourly wage rate for overtime work is less than the hourly wage for standard hours. We also assume that the premium for overtime work is regulated by a government and is taken as given.

The results obtained from this paper are as follows: (i) a reduction in standard hours can decrease unemployment, provided that overtime work is not fully compensated in the economy. Accordingly, this paper indicates that the policy of working time reduction will increase the number of vacancies subject to some conditions; (ii) unemployment can be also reduced by forcing firms to give workers more remuneration for overtime work, given that standard hours are fixed; (iii) a working time reduction policy may increase the ratio of higher-paying jobs, as well as reduce unemployment when the separation rate is low (e.g., the economy is in a boom). We must note that the above results will hold for situations in which unemployed workers are poorly compensated. The results of this paper suggest that a policy of reducing standard hours will increase employment in labor markets similar to that of Japan. In addition, this reduction policy will also have a favorable effect on the dispersion of wages in that the ratio of firms that post higher wages will rise because the market becomes more competitive for employers than before, due to the reduction in standard hours.

The paper is organized as follows. In Section 2, we explain the basic structure of the model, including a characterization of the matching technology. In Section 3, we describe the job acceptance behavior of workers and specify the expression of the reservation wage. In Section 4, we characterize a wage offer distribution and a market equilibrium. In Section 5, we examine the impact that a reduction in standard hours has on unemployment, and we show that a working time reduction policy will increase the proportion of high-paying jobs. Finally, in Section 6, we offer our conclusions.

2 Basic Structure of the Model

2.1 Matching Technology

There are many identical workers and firms in the economy. The measure of total labor force is denoted by one. Workers are either employed or unemployed. Let u denote the proportion of unemployed workers and v denote the proportion of vacancy relative to the total labor force. In this model, both unemployed and those who are employed workers engage in search activity. Furthermore, vacant firms post wages and seek trading partners. A matching function $m(v,\, u,\, 1-u)$ indicates that the number of worker−firm pairs realized in the search process depends on the vacancy rate, the unemployment rate and employment rate. We suppose that this matching function is increasing with each component and exhibits constant returns to scale, as described in Mortensen and Pissarides (1999). Unemployed and employed workers are equally productive, and they are perfectly substitutable not only as a component of the matching function but also as an input of the production function. These workers receive job offers at the rate $\mu_0(v)$ and $\mu_1(v)$, respectively. Unemployed workers accept an offer if it exceeds their reservation wage at which a worker is indifferent to being unemployed or being employed. In this regard, an offer is a random variable with a probability distribution, and the probability that workers meet some offer depends on the shape of this distribution.

The number of matches can be expressed as $m(v, u, 1-u) = \mu_0(v)\, u + \mu_1(v)\,(1-u)$. Below, we assume that unemployed workers and employees face the same arrival rate of job offers, that is, $\mu_0(v) = \mu_1(v)$. Denoting $\mu_0(v) = \mu_1(v) \equiv \lambda(v)$. Then $\lambda(v)$ is the job finding rate for workers, and $\lambda(v)/v$ is the arrival rate of meeting workers for firms with a vacant job. We assume that $\lambda(v)$ is a concave function, and its limit has the following properties:

$$\lim_{v\to 0} \lambda(v) = \lim_{v\to\infty} \frac{\lambda(v)}{v} = 0, \quad \lim_{v\to+\infty} \lambda(v) = \lim_{v\to 0} \frac{\lambda(v)}{v} = +\infty.$$

Job seekers find jobs more easily when there is an increase in vacancies, while vacant firms cannot easily find workers. Similarly, where there are fewer vacancies than before, vacant jobs find workers more easily, while job seekers cannot easily find jobs. The above assumptions indicate that economic agents in a search process face trading externalities.

2.2 Compensation for Workers

In this model, firms decide not only wage payments but also working time (they do so intuitively, by determining the number of tasks assigned to an employee). [4] Let l be the working time that is a firm's choice variable, and let $\bar{l}$ be the standard hours determined by the law, such as the Labor Standards Law in Japan (40 hours per week). Strictly speaking, this is not a statutory number of hours worked, but we treat it as an exogenous policy parameter below (see Brunello (1989)). If firms make employees work more than this number of hours, they must pay an overtime premium on the excess time $l - \bar{l}$ (in many countries, this premium must be at least 25% greater than the normal hourly wage rate). However, we define the compensation for workers as follows. First, employers pay a wage w for $\bar{l}$-time work. This w is not an hourly wage rate but the payment for a certain specified work time. Second, let $\beta\, w\, /\, \bar{l}$ be the hourly wage rate for overtime work, and β is assumed to be exogenous. [5] Later, we provide a detailed explanation of β.

We assume that w is independent of standard hours because it is difficult, for example, for employers to grasp the performance of white-collar workers compared to blue-collar workers. In general, white-collar workers are considered to be engaging in more complex tasks than blue-collar workers. In such situations, it is not reasonable for employers to

4) One might assume that workers determine their own work time. In reality, their decision regarding work time depends on the number of tasks assigned to them. Therefore, we treat the situation as though employers directly determine the number of hours their employees must work.

5) We regard the hourly wage rate for overtime work as a means of regulation for a government that should be taken as given for employers and workers. In this context, as pointed out in Hunt and Katz (1998), overtime premiums are legislated in countries such as the U.S. and France and they are negotiated in the U.K.

associate the value of the employees' work with their working time, and therefore employers are not compelled to compensate employees based on an hourly wage. The literature indicates that white-collar workers feel pressure from too much overtime work. Mizunoya (2005) demonstrates that the annual working time in the manufacturing sector will be less than the average annual working time in all industries (the difference is about 400 hours in Japan). [6)] This would suggest that white-collar workers will suffer more from overtime work than blue-collar workers (according to the British case illustrated in Hart (2004), about 50% of managers and professionals work overtime without pay, while only about 5% of plant and machine operatives work unpaid overtime). It follows that overtime work is of great importance from white-collar workers' points of view, and thus we conclude that the normal wage w is independent of standard hours.

Concerning payment for overtime work, Mizunoya (2005) points out that unpaid overtime is a prevailing phenomenon in many developed countries: (i) the level of overtime with pay is estimated as 143 hours in Japan (1993), 198 hours in the U.S.A. (1993), 127 hours in the U.K. (1993), 44 hours in Germany (1993), and 46 hours in Canada (1997); (ii) the level of unpaid overtime is estimated as 270 hours in Japan (1993), 116 hours in the U.S.A, (1993), 89 hours in the U.K. (1993), 26 hours in Germany (1993), and 59 hours in Canada (1997). [7)] These results indicate that workers in Japan perform the highest total number of hours of overtime work and that the ratio of unpaid overtime to the total hours of overtime worked is also highest in Japan. However, the most important fact is that, to varying degrees, workers in many countries work overtime, and we recognize that governments in these countries are implicitly permitting a situation in which the rule of compensation for overtime is frequently violated. [8)] Therefore, it is natural to assume that $\beta < 1$ (that is, we assume that the hourly wage rate for overtime work does not even reach the rate for standard hours).

Based on this assumption, we can calculate the actual value of β by using the data referred to above. For example, the value of β obtained from the Japanese data is computed as

$$\beta_{\text{Japan}} \times W \times 413 = 1.25 \times W \times 143 \Rightarrow \beta_{\text{Japan}} = \frac{1.25 \times 143}{413} \cong 0.43,$$

where w is an hourly wage rate, and 1.25 is the minimum premium rate for overtime work in Japan. If overtime work were fully compensated, β_{Japan} would be close to 1.25. As we show, however, the value of β_{Japan} is far below this. For other countries, the actual value of β is also less than 1 and, moreover, below the countries' respective overtime premium rates. [9)]

6) Similar circumstances are observed in the U.S., the U.K., France, and Germany. See Mizunoya (2005).

7) The hours shown above correspond to annual working hours for full-time male workers. Female workers in Japan also work the most overtime among those countries. These instances support the assumption that $\beta < 1$.

8) Contensou and Vranceanu (2000) illustrate the basic elements of regulation imposed on a compensating system in various countries. See Contensou and Vranceanu (2000). Apart from Mizunoya (2005), little research has been performed comparing the level of unpaid overtime worked in various countries.

9) Bell and Hart (1999*a*) find that the average overtime premium in the unregulated British labor market is 1.4; in addition, they estimate that the premium is independent of the amount of overtime work. Here, the term "unregulated market" is defined as one in which overtime and the premium for this additional time are not regulated by law.

When actual hours worked are l and the wage for the standard hours is w, the total labor income is

$$\text{Total income} \;=\; \begin{cases} w + (l-\bar{l})\beta\, w\,/\,\bar{l} & \text{if } \; l > \bar{l}, \\ w & \text{if } \; l \leq \bar{l}, \end{cases}$$

The flow utility, denoted by x for each period, can be written as

$$x(w,\, l) \;=\; w \;+\; \beta\, w \,\max\left\{\frac{(l-\bar{l})}{\bar{l}},\, 0\right\} \;-\; d(l), \tag{1}$$

where $d(l)$ is a disutility function and is given as $d(l) = l^2$ for simplicity below. Employed workers receive $x(w,\, l)$ for every period when they get w and work l hours. In this model, we suppose that employees must work at least $\bar{l}$. Therefore, they receive compensation for overtime work that is expressed as $(l - \bar{l})\, \beta\, w/\bar{l}$.

2.3 The Value of Workers

The Bellman equation for unemployed workers, J_U, is

$$r\, J_U \;=\; b + \lambda(v)\left[\int_{w_R}^{\bar{w}} J_E(w,\, l)\, d\Gamma(w) - J_U\right], \tag{2}$$

where b is a benefit for unemployment received by all jobless workers, r is the discount rate, and $\Gamma(\cdot)$ is the wage offer distribution. In addition, $J_E(w,\, l)$ is the value of employment when employees receive the normal wage w and work l hours. $\bar{w}$ is the supremum of the support of $\Gamma(\cdot)$, and w_R is the reservation wage. We first assume the existence of the distribution $\Gamma(\cdot)$ and specify its concrete shape by solving the firm's profit maximization problem. Since unemployed workers never accept an offer that is less than the reservation wage, this is the lowest element in the support of the wage offer distribution. Thus, job seekers will accept any wage offer in equilibrium.

The value of employed workers receiving w' and working l' hours, $J_E(w',\, l')$, is

$$\begin{aligned} r\, J_E(w',\, l') \;=\;& w' \;+\; \beta\, w' \,\max\left\{\frac{(l'-\bar{l})}{\bar{l}},\, 0\right\} \;-\; (l')^2 + \delta\,[\, J_U - J_E(w',\, l')\,] \\ &+ \lambda(v)\left[\int_{w'}^{\bar{w}} J_E(w,\, l)\, d\Gamma(w) + \int_{w_R}^{w'} J_E(w',\, l')\, d\Gamma(w) - J_E(w',\, l')\right], \end{aligned} \tag{3}$$

where δ is the exogenous separation rate of a match. In expression (3), we suppose that the higher wages provide workers with higher utility. As a result, when they receive an offer greater than w', they accept it and leave their current job. Job seekers in this model, however, make their job acceptance decision comprehensively by considering the normal wage, payment for overtime work, and the disutility suffered from work. In this context, we have to consider the possibility that the worker's utility is not an increasing function with respect to w' in (3), since hours worked, which are decided by firms, have an effect on this utility. We show, however, that the utility increases with w' when β is sufficiently low, that is, when firms are reluctant to pay for overtime work. This situation is consistent with the current condition of the labor market in Japan. We concentrate on this situation, and examine the effect that a policy of reducing standard hours has on unemployment.

2.4 The Value of Firms and the Job Creation Condition

We adopt the equilibrium search framework combined with the wage-posting model developed in Mortensen and Pissarides (1999), Rosholm (2000) and Albrecht and Vroman (2005). In this context, the state of firms is either vacant or filled. The value of a vacant job, J_V, is

$$r\,J_V \;=\; \max_{\{w,\,l\}} \left\{ \frac{\lambda(v)}{v}\,[\,u + (1-u)\,G(w)\,]\,(J_F(w,\,l) - J_V) - c \right\}, \tag{4}$$

where c is the maintenance cost, and $G(w)$ is the distribution of workers who obtain a wage less than or equal to w. $J_F(w,\,l)$ is the value of a filled job when the wage w is paid and an employee works l hours. Note that $u + (1-u)\,G(w)$ is the ratio of workers accepting the offer $w(\geq w_R)$ provided by firms, since all unemployed workers and employed workers who receive wages less than w accept this offer.

We suppose that the production technology of firms is given as exponential, and that labor is the only input. When a firm pays the normal wage w, the flow profit of the firm is

$$\pi(w,\,l) \;=\; l^{\gamma} \;-\; \left[w \;+\; \beta\,w\,\max\left\{ \frac{(l-\bar{l})}{\bar{l}},\,0 \right\} \right], \tag{5}$$

where the first term is the output generated from l hours worked and $\gamma \in (0,\,1)$. The objective of a firm is to maximize its expected profit with respect to the working time and the wage w. We first derive an optimal working time that maximizes (5), then describe it as a function of the normal wage. After that, we consider the maximization problem of J_V with respect to w. On the one hand, a higher wage attracts many applicants, but on the other, it reduces the flow profit for filling a job. Thus, there exists a trade-off between attracting workers and obtaining large flow profits, and as a consequence there are many wage offers that can act as solutions to this maximization problem. We characterize the wage offer distribution by using the optimal working time determined below.

For any wage level w, firms obtain the profit $\pi(w,\,l)$ by hiring one worker, which leads to the following first order condition with respect to l:

$$\gamma\,l^{\gamma-1} \;=\; \frac{\beta\,w}{\bar{l}} \quad\Rightarrow\quad l(w) \;\equiv\; \left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{1/(1-\gamma)}, \tag{6}$$

where we suppose that the optimum occurs in the region $l > \bar{l}$. This implies that $l(w)$ is an interior solution. It follows from (6) that $\partial l(w)/\partial \bar{l} > 0$ and $\partial l(w)/\partial \beta < 0$. The former result means that an increase in standard hours enables firms to hire workers at a lower wage because w is fixed and $\bar{l}$ is raised. For the latter result, firms cut hours worked as the overtime premium β increases, since they must pay higher compensation for overtime work than before.

The Bellman equation for filled jobs paying some w', $J_F(w')$, is

$$r\,J_F(w') \;=\; l(w')^{\gamma} - w' \left[(1-\beta) + \frac{\beta\,l(w')}{\bar{l}} \right] + \{\,\delta + \lambda(v)\,[\,1 - \Gamma(w')\,]\,\}\,(J_V - J_F(w')), \tag{7}$$

where $J_F(w') \equiv J_F(w',\, l(w'))$. Since this model permits on-the-job searches, the separation rate of a match depends on the offer arrival rate for workers and the shape of the wage offer distribution. That is, workers who receive an offer greater than w' leave their current job, and that job then becomes vacant.

An evaluation of $J_V = 0$ (the free entry/exit condition), (7) can be written as

$$J_F(w') \;=\; \frac{l(w')^{\gamma} - w' \,\left[\,(1-\beta) + \beta\, l(w')\,/\,\bar{l}\,\right]}{r + \delta + \lambda(v)\,[\,1 - \Gamma(w')\,]}. \tag{8}$$

Inserting this into (4) and rearranging it yields

$$\frac{c\,v}{\lambda(v)} \;=\; \max_{w'} \left\{ [\,u + (1-u)\,G(w')\,] \left[\frac{l(w')^{\gamma} - w\,\left[\,(1-\beta) + \beta\, l(w')\,/\,\bar{l}\,\right]}{r + \delta + \lambda(v)\,[\,1 - \Gamma(w')\,]} \right] \right\}. \tag{9}$$

This is the job creation condition that determines the number of vacant firms in the long-run equilibrium. This condition means that the discounted expected costs of having a vacancy are equal to the maximized future profits with respect to the normal wage. The substance of this condition is the same as in the Mortensen and Pissarides (1999) and Rosholm (2000). After we specify the wage offer distribution $\Gamma(\cdot)$, the vacancy rate satisfying this condition can be discussed.

3 Job Acceptance Decision for Workers

The model developed in this paper is based on the Burdett-Mortensen wage-posting model with a matching framework. However, if the flow utility of a worker does not increase with the normal wage w, making higher wage offers will no longer be the optimal behavior of employers. One reason the reservation wage is characterized uniquely in the Burdett-Mortensen framework is that the expected lifetime utility of workers increases with their received wages. In order to use the same procedures as in the standard model, we must show that this monotonicity is still maintained even when overtime work and the disutility of work are introduced into the model. [10)]

It follows from (3) that the monotonicity of the worker's lifetime utility is replaced by that of the flow utility $x(w)$. The derivative of $x(w)$ with respect to w is

$$\frac{d\,x}{d\,w} \;=\; 1 - \beta + \frac{\beta}{\bar{l}\,(1-\gamma)} \left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} \left[\frac{2}{\gamma} \left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{(2-\gamma)/(1-\gamma)} - \gamma \right]. \tag{10}$$

Then, a sufficient condition for (10) being positive is

$$\beta \;<\; \left(\frac{\bar{l}\,\gamma}{w}\right) \left(\frac{\gamma^2}{2}\right)^{-(1-\gamma)\,/\,(2-\gamma)} \qquad \text{for all } w. \tag{11}$$

10) Hwang et al. (1998) extend the Burdett-Mortensen model to allow employers to determine a non-monetary job amenity as well as a wage offer. In this case, job seekers accept an offer when the offer they receive provides greater utility than their reservation utility level, rather than their reservation wage. In that model, since the optimal amenity level for employers is independent of the wage payments they offer, the equilibrium utility distribution can be derived from procedures similar to those used in the Burdett-Mortensen model.

This is the condition for the value of β. Since the right hand side of (11) is decreasing in w, we must note that this inequality is difficult to satisfy at $w = \bar{w}$. Furthermore, if the value on the right side at $w = \bar{w}$ is greater than 1, condition (11) does not constrain the value of β. However, we cannot ascertain whether the condition is constrained or not. Thus, (11) must be taken into account in subsequent studies.

Additionally, we must consider the assumption that the optimal working time $l(w)$ is greater than $\bar{l}$. It follows from (6) that this condition is expressed by

$$\beta < \bar{l}^{\gamma} \left(\frac{\gamma}{w}\right), \quad \text{for all } w.$$

This also requires that β be sufficiently low. The condition for β is thus given by

$$\beta < \min \left[1,\ \bar{l}^{\gamma} \left(\frac{\gamma}{w}\right),\ \left(\frac{\bar{l}\,\gamma}{w}\right) \left(\frac{\gamma^2}{2}\right)^{-(1-\gamma)\,/\,(2-\gamma)} \right], \quad \text{for all } w. \tag{11$'$}$$

Condition (11′) indicates that the monotonicity of x with respect to w requires a sufficiently low overtime premium. Since a higher w results in a lower actual work time in (6), the disutility of work declines. On the other hand, this reduction also decreases the income received from overtime work in spite of the increase in the hourly wage rate, $\beta\, w/\bar{l}$. This is because a reduction in $l(w)$ results in fewer overtime hours. When β is low, a decrease in overtime compensation has little effect on the worker's utility. This makes (10) positive; the increase in utility due to the reduction in $l(w)$ has a greater impact on $x(w)$. As pointed out in Mizunoya (2005), unpaid overtime in Japan is greater relative to other developed countries. This corresponds to a case in which β is sufficiently low. Because of this, we can assume that β satisfies condition (11′). In this case, since the monotonicity of $J_E(w) \equiv J_E(w, l(w))$ with respect to w is assured, the reservation wage is uniquely determined, as in Mortensen and Pissarides (1999) and other related works.

It follows from (2), (3) and the definition of the reservation wage property, $J_E(w_R) = J_U$, that w_R is the solution to the following equation:

$$\gamma \left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{\gamma/(1-\gamma)} + (1-\beta)\, w_R - \left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{2/(1-\gamma)} = b. \tag{12}$$

Since the left side of (12) is increasing with w_R because $x'(w) > 0$ for all w, this condition has a unique solution. Furthermore, we can obtain the following results from (12):

$$\frac{\partial w_R}{\partial b} > 0, \qquad \frac{\partial w_R}{\partial \bar{l}} > 0.$$

The first result is intuitive, [11)] and the latter finding results from condition (11′). It follows from (6) that longer standard hours lead to a longer actual working time. Because of this, workers will demand more income as work time increases, since they suffer greater disutility. From the firm side, to compensate employees for greater work time, employers must pay wages high enough that workers have an incentive to be employed.

11) Albrecht and Vroman (2005) show that the reservation wage decreases with unemployment benefits when an economy has a two-tier unemployment compensation system.

4 Wage Offer Distribution

4.1 The Flow of Workers

The unemployment rate in the steady state occurs at the point where the flow of workers who leave the unemployment pool equals the flow into the pool. This inflow and outflow can be described by $(1-u)\,\delta$ and $u\,\lambda(v)\,[\,1-\Gamma(w_R)\,]$, respectively, where δ is the exogenous separation rate of a match and $\lambda(v)\,[\,1-\Gamma(w_R)\,]$ is the arrival rate of job offers with wages greater than w_R. Stationarity requires that these flows should be equal. This results in the following expressions:

$$u = \frac{\delta}{\delta + \lambda(v)\,[\,1-\Gamma(w_R)\,]}, \tag{13}$$

$$1-u = \frac{\lambda(v)\,[\,1-\Gamma(w_R)\,]}{\delta + \lambda(v)\,[\,1-\Gamma(w_R)\,]}, \tag{14}$$

where (14) represents the employment rate.

We denote the proportion of employed workers receiving a wage less than or equal to w as $G(w)$. Transitions of such worker flows are then given by

$$\begin{aligned} \frac{d\,[\,1-u(t)\,]\,G(w)}{dt} &= u(t)\,\lambda(v)\,[\,\Gamma(w)-\Gamma(w_R)\,] - [\,1-u(t)\,]\,G(w) \\ &\quad \times \{\delta + \lambda(v)\,[\,1-\Gamma(w)+\Gamma(w_R)\,]\,\}\,. \end{aligned}$$

The first term on the right hand side is the inflow into the employment pool of workers who receive at most w, and the latter term is the outflow of employed workers who find a better job or are fired. Evaluating this expression in the steady state results in

$$G(w) = \frac{\delta\,[\,\Gamma(w)-\Gamma(w_R)\,]}{[1-\Gamma(w_R)\,]\,\{\delta + \lambda(v)\,[\,1-\Gamma(w)+\Gamma(w_R)\,]\,\}}. \tag{15}$$

This is the steady state value of $G(w)$. Using (15) to calculate $u+(1-u)\,G(w)$ yields

$$u + (1-u)\,G(w) = \frac{\delta}{\delta + \lambda(v)\,[\,1-\Gamma(w)\,]}. \tag{16}$$

This is the proportion of workers who will accept the offer w when a vacant firm posts this wage. Since employers know that workers never accept an offer that is strictly less than w_R, they must provide at least this reservation wage level. This implies that $\Gamma(w_R) = 0$.

Substituting (16) into (9) yields

$$\frac{c\,v}{\lambda(v)} = \max_{w} \left\{ \frac{\delta\,\left[\,\left(\bar{l}\,\gamma/\beta\,w\right)^{\gamma/(1-\gamma)} - (1-\beta)\,w - \gamma\,\left(\gamma\,\bar{l}/\beta\,w\right)^{\gamma/(1-\gamma)}\,\right]}{\{\,\delta + \lambda(v)\,[\,1-\Gamma(w)\,]\,\}^2} \right\}, \tag{17}$$

where we assume $r \to 0$ for simplicity. In the following section, we characterize the wage offer distribution, and describe the job creation condition (17) as the relationship between the vacancy rate and market parameters.

4.2 Description of Wage Offer Distribution

Here we derive a wage offer distribution that characterizes wage dispersion in a steady state equilibrium. It follows from the argument in the standard Burdett-Mortensen model that the equivalence of profits, obtained by posting any offer contained in $[\,w_R,\, \bar{w}\,]$, gives a concrete shape to the distribution. This equivalence condition requires that the profit obtained from offering the reservation wage be equal to the profit obtained by posting any offer of the support $[\,w_R,\, \bar{w}\,]$. We must note that posting lower wages generates higher flow profits, but such offers cannot attract workers in the search process, and a firm will take a long time to fill its vacancy. On the other hand, posting higher wages results in lower flow profits while attracting many applicants. Thus, it is possible that numerous wage offers generate the same profits. This profit equivalence condition requires that for all $w \in [\,w_R,\, \bar{w}]$,

$$\frac{(1-\gamma)\,\left(\bar{l}\,\gamma/\beta\, w\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w}{\{\,\delta + \lambda(v)\,[\,1-\Gamma(w)\,]\,\}^2} \;=\; \frac{(1-\gamma)\left(\bar{l}\,\gamma/\beta\, w_R\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w_R}{[\,\delta +\, \lambda(v)\,]^2}, \tag{18}$$

where the expected profits of a firm from posting w are given as the right side of (17). By solving for $\Gamma(w)$, condition (18) yields the following expression:

$$\Gamma(w) \;=\; \frac{\delta +\, \lambda(v)}{\lambda(v)} \left[\, 1 - \sqrt{\frac{(1-\gamma)\,\left(\bar{l}\,\gamma/\beta\, w\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w}{(1-\gamma)\left(\bar{l}\,\gamma/\beta\, w_R\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w_R}}\,\right]. \tag{19}$$

Expression (19) represents the relationship between $\Gamma(w)$ and w. Since the supremum of the support must satisfy $\Gamma(\bar{w}) = 1$, the condition that characterizes $\bar{w}$ is given by

$$\begin{aligned} &(1-\gamma)\,\left(\frac{\bar{l}\,\gamma}{\beta\,\bar{w}}\right)^{\gamma/(1-\gamma)} - \bar{w}\,(1-\beta) \\ &\quad = \left(\frac{\delta}{\delta + \lambda(v)}\right)^2 \left[\,(1-\gamma)\,\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w_R\,\right]. \end{aligned} \tag{20}$$

Note that as the left side of (20) decreases with the increase of $\bar{w}$ and the right side is constant, [12] there exists a unique $\bar{w}$ satisfying (20). The discussion in this subsection means that, given any v, the unique distribution $\Gamma(\cdot)$ is determined by (12), (19) and (20).

5 Steady State Equilibrium

5.1 Job Creation Condition

In Burdett and Mortensen (1998), the arrival rate of job offers is given as constant. In our model, however, the arrival rate depends on the matching technology and, therefore on

12) In addition, the left side of (20) tends to $+\infty$ as $\bar{w} \to 0$, and it tends to $-\infty$ as $\bar{w} \to \infty$.

the vacancy rate. The vacancy rate in the steady state is determined by the job creation condition, [13] which is obtained from (17) at $w = w_R$:

$$\frac{c\,v}{\lambda(v)} = \frac{\delta}{[\,\delta + \lambda(v)\,]^2} \left[(1-\gamma) \left(\frac{\bar{l}\,\gamma}{\beta\, w_R} \right)^{\gamma/(1-\gamma)} - (1-\beta)\, w_R \right] . \tag{21}$$

Since employers can obtain the same profits by offering any w in $[\,w_R,\, \bar{w}\,]$, w_R must be one of the solutions that maximizes the expected profits of a firm. Thus, the right side of (17) is replaced by (21).

We can show the existence of v satisfying (21) as follows. The right side of (21) is decreasing with respect to v, and its left hand side is Increasing, since the following property is satisfied:

$$\lambda(v) - v\,\lambda'(v) \;=\; v\,\left(\frac{\lambda(v)}{v} - \lambda'(v) \right) > 0.$$

This results from the assumption that $\lambda(v)$ is concave. Since the left side of (21) represents the expected costs for a vacant firm and the right side of the expression represents its expected profits, an increase in v means that vacant firms suffer from large expected costs because the labor market becomes competitive. On the other hand, this increase in v reduces expected profits because employed workers are more likely to leave their current jobs. These results ensure that a unique vacancy rate satisfying (21) exists. [14]

Condition (21) uniquely determines the vacancy rate in the steady state equilibrium. This unique vacancy rate fixes the unemployment rate in the equilibrium by (13). Thus equilibrium in this model is composed of w_R, u, $\Gamma(\cdot)$, $\bar{w}$,and v and is completely described by (12), (13), (19), (20), and (21).

Proposition 1
The vacancy rate satisfying (21) *is uniquely determined. Therefore, a unique equilibrium is also characterized by* (12), (13), (19), (20), *and* (21).

5.2 The Effect of Reducing the Standard Hours on Unemployment

Policy makers and economists have great interest in whether a reduction in standard hours increases employment. We provide one possible answer to this question by using a model based on the wage-posting framework combined with a matching model. The result we obtain is different from the one obtained using a bargaining-based model, as in Marimon and Zilibotti (2000). In order to show this, we must examine the effect of a change in $\bar{l}$ on the equilibrium unemployment rate.

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13) If a match is destructed endogenously through a random shock, it requires another condition (the job destruction condition) to characterize the equilibrium. See Mortensen and Pissarides (1994) and Pissarides (2000).

14) We note that if v goes to zero, the LHS of (21) also goes to zero and its RHS takes some finite value. Furthermore, if v goes to $+\infty$, the LHS of (21) also goes to $+\infty$ and its RHS goes to zero. Thus there exists some v that satisfies the condition (21).

It follows from (13) that the steady state unemployment rate decreases with v and does not depend on other endogenous variables or $\bar{l}$. This means that a reduction in the standard hours has an effect on unemployment only through the vacancy rate. Thus, it suffices to examine the sign of $\partial v/\partial \bar{l}$ from (21). This is equal in sign to

$$-\frac{\beta\, w_R}{\bar{l}^2}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)}\left(\frac{\bar{l}}{w_R}\frac{\partial w_R}{\partial \bar{l}}-1\right)-(1-\beta)\,\frac{\partial w_R}{\partial \bar{l}}. \tag{22}$$

From (13) and (21), the following facts are derived:

$$\begin{aligned}(22)\ >0 \quad &\Rightarrow \quad \frac{\partial v}{\partial \bar{l}}\ >\ 0 \ \text{ and } \ \frac{\partial u}{\partial \bar{l}}\ <\ 0,\\ (22)\ <0 \quad &\Rightarrow \quad \frac{\partial v}{\partial \bar{l}}\ <\ 0 \ \text{ and } \ \frac{\partial u}{\partial \bar{l}}\ >\ 0.\end{aligned}$$

That is, if a reduction in $\bar{l}$ raises the flow profits of a filled job, then the amount of vacancy in the economy will increase. In short, the effect that reducing the standard hours has on unemployment depends upon the sign of (22). In the following, we specify the condition for the case in which (22) < 0.

We consider the pairs $(\bar{l},\, \beta)$ obtained by making the transformed version of (22) equal zero (Appendix A exhibits the results of $\partial w_R/\partial \bar{l}$ and $\partial(w_R/\bar{l})/\partial\, \bar{l}$ that are necessary to derive the following equation). These pairs satisfy

$$\frac{\gamma\,(1-\beta)}{(1-\gamma)\,\bar{l}\,\Delta}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{\gamma/(1-\gamma)}\left[1-\frac{2}{\gamma}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)}\right]\ =\ 0, \tag{23}$$

where

$$\Delta\ \equiv\ 1-\beta+\frac{\beta\,\gamma}{(1-\gamma)\,\bar{l}}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)}\left[\frac{2}{\gamma^2}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)}-1\right]>0.$$

Rearranging (23) yields

$$\left(\frac{\gamma}{2}\right)^{-(1-\gamma)/(2-\gamma)}\ =\ \frac{\beta\, w_R}{\bar{l}\,\gamma}. \tag{24}$$

This condition specifies the area of pairs $(\bar{l},\, \beta)$ that realizes an increase in v by reducing $\bar{l}$. We note that w_R is dependent on $\bar{l}$ and β:

$$\frac{\partial\,(\beta\, w_R)}{\partial\,\beta}\ >\ 0 \ \text{ and } \ \frac{\partial}{\partial\,\bar{l}}\left(\frac{w_R}{\bar{l}}\right)\ <\ 0.$$

Thus, the right side of (24) is increasing in β and decreasing in $\bar{l}$. Accordingly, we can rewrite (24) as $\beta=\phi(\bar{l})$, and find that $\phi(\cdot)$ is an increasing function of $\bar{l}$.

Figure 1 (left) stands for (24) in $(\bar{l},\, \beta)$-plane, and (11′) and (24) are depicted on the same plane on the right side of the figure. Although it is difficult to specify the concrete shape of condition (11′), we know that this constraint requires that β be sufficiently low for any given $\bar{l}$. Then, $\beta=\phi(\bar{l})$ separates the area that is created by (11′), the vertical axis and the horizontal axis into two parts. These areas are denoted as Area A and Area B, respectively. They are defined as follows:

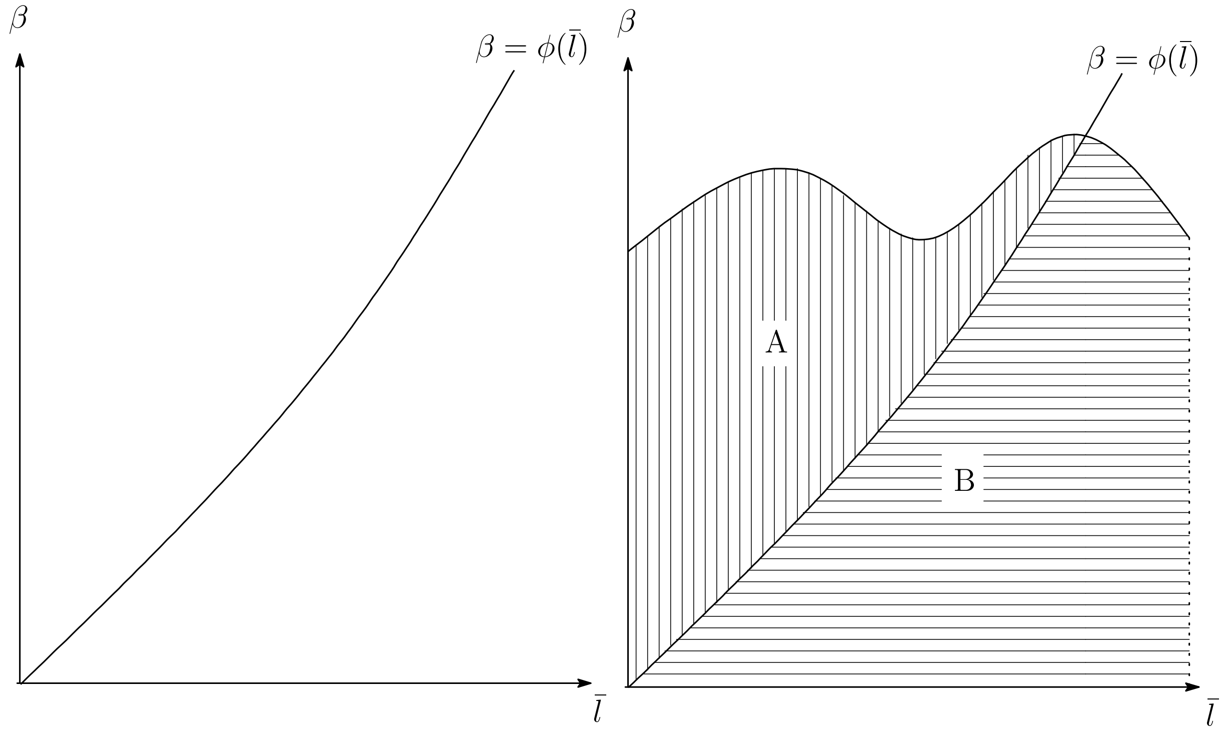

Figure 1: Description of (24) and the Impact of the Reduction in Working Time

(Area A): For a given $\bar{l}$, β takes a higher value than the one satisfying (24) in this area. The left side of (23) has a positive sign, entailing that (22) is positive. Then, we obtain $\partial v/\partial \bar{l} > 0$ and $\partial u/\partial \bar{l} < 0$. Therefore, in this case, a reduction in the standard hours increases unemployment.

(Area B): For a given $\bar{l}$, β takes a lower value than the one satisfying (24) in this area. The left side of (23) has a negative sign, which means that (22) is negative. Then, we obtain $\partial v/\partial \bar{l} < 0$ and $\partial u/\partial \bar{l} > 0$. Therefore, in this case, a reduction in the standard hours reduces unemployment.

In this context, we must note that condition (11′) evaluated at $w = \bar{w}$ becomes a complex figure. This is because β has not only a direct effect on $\bar{w}$, but also an indirect effect on $\bar{w}$ through the endogenous variables v and w_R. Nevertheless, it is difficult to show the existence of such a β analytically. Therefore, we cannot specify whether the right side of (11′) is a monotone function of β. Consequently, we focus on the fact that a lower value of b (unemployment compensation) increases the right side of (11′), since $\partial w_R/\partial b > 0$ and $\partial \bar{w}/\partial w_R > 0$ by (12) and (20) and the left side of (11′) is obviously independent of b. Thus, lower β is likely to satisfy (11′) even at $w = \bar{w}$ for a sufficiently low b; this reflects the situation in which unemployed workers are poorly compensated. We suppose that b is small enough that the condition (11′) is satisfied for a low β.

In the end, we obtain the following proposition.

Proposition 2

A reduction in standard hours results in a lower unemployment rate when β is low, that *is, when unpaid overtime work is prevalent in the economy.*

Proposition 2 indicates that reducing the standard hours will increase employment when employed workers customarily engage in unpaid overtime work. We can explain this proposition as follows. Employers have an incentive to make their employees work longer as β takes a lower value. In this case, the marginal product of work time is smaller than the marginal product when β is high, since the output per employee is given by an exponential function with a quotient that is less than one. Because of this, the degree of the reduction in revenue due to declining work hours is small, and employers will acquire the benefits from this reduction policy. Accordingly, the reduction in $\bar{l}$ yields more profits, providing employers with an incentive to create more jobs in the economy. This reduces the number of unemployed workers because they have more opportunities for employment than before.

5.3 The Effect on Unemployment of a Rise in the Hourly Wage Rate for Overtime Work

Next, we investigate the impact that a change in β has on unemployment. In Proposition 2, we show that the effectiveness of a work time reduction depends on the value of β. However, we have not examined the direct effect of an increase in payment for overtime work. This is another topic of interest.

The partial derivative of (21) with respect to β is equal in sign to

$$-\frac{1}{\bar{l}}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)}\left(w_R+\beta\,\frac{\partial w_R}{\partial\beta}\right)-(1-\beta)\,\frac{\partial w_R}{\partial\beta}+w_R.$$

This expression becomes

$$w_R\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)}\left[\frac{2}{\gamma}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)}-1\right]$$
$$\Bigg/\ \bar{l}\,(1-\gamma)\left[\frac{\beta\,\gamma}{\bar{l}(1-\gamma)}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)}+(1-\beta)+\frac{2\,\beta}{\gamma\,\bar{l}(1-\gamma)}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(3-\gamma)/(1-\gamma)}\right].$$

Since we already know that the denominator of this expression is positive for a sufficiently low β, the overall sign depends on the sign of its numerator. Suppose that β is small enough that a reduction in $\bar{l}$ decreases unemployment. It follows from (24) that this condition is expressed by

$$\frac{2}{\gamma}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)}-1>0. \tag{25}$$

It then follows that $\partial v/\partial\beta>0$, and therefore, forcing firms to give workers greater compensation for overtime work also expands the number of vacancies and decreases unemployment as long as (25) is satisfied.

Proposition 3
When condition (25) *is satisfied* (*this is the same as a situation in which a reduction in* $\bar{l}$ *results in a lower unemployment*), *an increase in* β *reduces unemployment.*

We can give the same explanation for this proposition as in Proposition 2. The result of Proposition 3 supports the conjecture made by Trejo (1991). That is, a government may stimulate the creation of jobs by tightening overtime pay regulation (expressed by an increase in β). Since $\partial\,(\beta\, w_R)\,/\,\partial\,\beta$ is positive, the rise in β results in fewer total hours worked, $l(w_R)$. This decrease in hours worked will contribute to expanding job creation.

Eventually, reducing the standard hours decreases unemployment when employers do not sufficiently compensate employees for their overtime work. Furthermore, in the same situation, unemployment can also be reduced by forcing firms to give workers greater compensation for overtime work when the standard hours are unchanged.

In Marimon and Zilibotti (2000), if additional compensation paid by firms for overtime work is transferred to the workers, reducing working time increases employment under a production technology with decreasing returns to labor, irrespective of the level of overtime premium and the standard hours. [15] In summary, so long as the surcharges on overtime are transferred to the workers, the existence of overtime work does not fundamentally change the main result of Marimon and Zilibotti's (2000) model. By contrast, the results of our model critically depend on the value of the parameters $\bar{l}$ and β as described in Figure 1. The introduction of overtime work has a great impact on the effectiveness of the working time reduction policy. Additionally, Marimon and Zilibotti (2000) construct the model such that the surcharges imposed on overtime do not depend on the hourly wage rate paid for standard hours. This simplifies the arguments but is not consistent with institutions in the real world. In contrast to Marimon and Zilibotti (2000), in our model, the wage rate paid for overtime work reflects both wage dispersion and the normal wage corresponding to standard hours. This mechanism accurately represents those institutions and enriches the model. Examining the effect of a reduction in working time on employment in such a framework is necessary when assessing the validity of such a policy.

5.4 The Effect of Reducing Standard Hours on the Dispersion of Wage Offers

In this subsection, we examine the effect that a reduction in $\bar{l}$ has on the wage offer density function. As has been shown, this policy reduces unemployment when β is sufficiently low. While this result seems to offer support for a policy of work time reduction, we have not studied its effect on the dispersion of wage offers. In the following analysis, we show that the ratio of low-paying jobs in the economy is likely to decline and that of high-paying jobs may increase in this scenario.

15) On the other hand, Marimon and Zilibotti (2000) show that, when surcharges for overtime are not transferred to the workers (i.e., these surcharges are regarded as higher taxes), reducing the standard hours decreases employment provided that the rate of surcharges is kept constant; also, employment increases when a sufficiently large rate of surcharges is imposed on the use of overtime work.

From (19), the wage offer density function is given by

$$\frac{d\,\Gamma(w)}{d\,w} = \frac{\delta+\lambda(v)}{2\,\lambda(v)\,\sqrt{(1-\gamma)(\bar{l}\,\gamma/\beta\,w_R)^{\gamma/(1-\gamma)}-(1-\beta)w_R}} \times \left[\frac{(\beta/\bar{l})(\bar{l}\gamma/\beta\,w)^{1/(1-\gamma)}+(1-\beta)}{\sqrt{(1-\gamma)(\bar{l}\,\gamma/\beta\,w)^{\gamma/(1-\gamma)}-(1-\beta)w}}\right]. \quad (26)$$

Here, we consider the case in which $\partial v/\partial \bar{l} < 0$. Thus, the first line of the above expression is increasing with $\bar{l}$ since

$$\frac{d}{d\,v}\left(\frac{\delta+\lambda(v)}{\lambda(v)}\right) > 0 \quad \text{and} \quad (1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta\,w_R}\right)^{\gamma/(1-\gamma)} - (1-\beta)w_R \;\text{ is decreasing in } \bar{l}.$$

On the other hand, the effect of $\bar{l}$ on the second line of (26) is ambiguous, and the result depends on the size of w (the derivation process of (27) is described in Appendix B):

$$\begin{aligned}
&\frac{\partial}{\partial \bar{l}}\left[\frac{(\beta/\bar{l})(\bar{l}\gamma/\beta\,w)^{1/(1-\gamma)}+(1-\beta)}{\sqrt{(1-\gamma)(\bar{l}\,\gamma/\beta\,w)^{\gamma/(1-\gamma)}-(1-\beta)\,w}}\right] \\
&= \frac{\gamma}{1-\gamma}\,\bar{l}^{(2\gamma-1)/(1-\gamma)}\left(\frac{\gamma}{\beta}\right)^{1/(1-\gamma)}\left[\frac{\beta\,(1-\gamma)}{2}\left(\frac{\bar{l}\,\gamma}{\beta}\right)^{\gamma/(1-\gamma)} w^{(\gamma+1)/(\gamma-1)}\right. \\
&\quad \left. - \frac{\beta\,(1-\beta)(1+\gamma)}{2\,\gamma}\,w^{\gamma/(\gamma-1)}\right] \Bigg/ \left[(1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{\gamma/(1-\gamma)} - (1-\beta)\,w\right]^{3/2}. \quad (27)
\end{aligned}$$

We focus on the term within the bracket in the numerator of (27) and rewrite it as follows:

$$\frac{\beta\,w^{-\gamma/(1-\gamma)}}{2\,\gamma}\left[\gamma\,(1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta}\right)^{\gamma/(1-\gamma)} w^{-1/(1-\gamma)} - (1-\beta)(1+\gamma)\right]. \quad (28)$$

From (28), we find that a lower (or higher) wage offer makes the sign of (28) positive (or negative). Concretely speaking, there exists a wage level $\hat{w}$ that makes (28) zero:

$$\hat{w} \equiv \left(\frac{\bar{l}\,\gamma}{\beta}\right)^{\gamma}\left[\frac{\gamma\,(1-\gamma)}{(1-\beta)(1+\gamma)}\right]^{1-\gamma}. \quad (29)$$

(29) indicates that for any $w > \hat{w}$, (27) becomes negative and for any $w < \hat{w}$ (27) becomes positive.

We notice that for any w that is less than $\hat{w}$, the density of such w declines with a reduction in standard hours. This implies that the policy of reducing standard hours decreases the number of low-paying jobs. On the other hand, for an offer that is greater than $\hat{w}$, it is difficult to conclude that its density unambiguously increases as $\bar{l}$ is reduced, since the former part of (26) is increasing in $\bar{l}$. [16] When there are many offers that exceed $\hat{w}$, however, reducing the standard hours will increase the density of these higher wages.

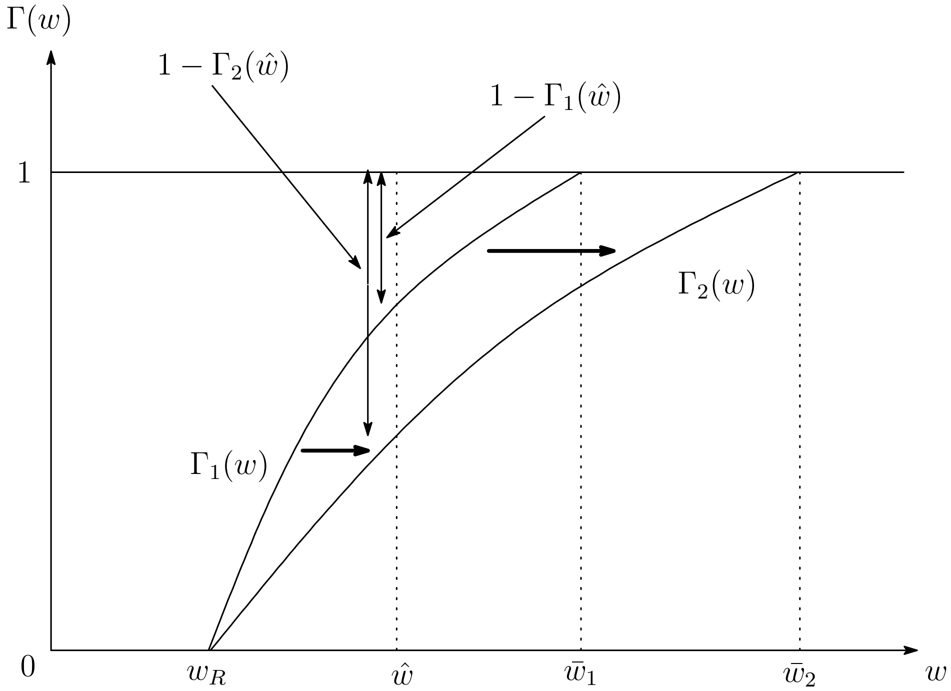

Figure 2: Γ_2 dominates Γ_1 by the first degree.

Below, we finally show that the probability $1 - \Gamma(\hat{w})$ is high when the separation rate δ is small.

From (19), $\Gamma(\hat{w})$ is given as

$$\Gamma(\hat{w}) = \frac{\delta + \lambda(v)}{\lambda(v)} \left[1 - \sqrt{\frac{(1-\gamma)\,\left(\bar{l}\,\gamma/\beta\,\hat{w}\right)^{\gamma/(1-\gamma)} - (1-\beta)\,\hat{w}}{(1-\gamma)\left(\bar{l}\,\gamma/\beta\,w_R\right)^{\gamma/(1-\gamma)} - (1-\beta)\,w_R}} \right].$$

Since $\hat{w}$ is independent of δ, it suffices to examine the reaction of $(\delta + \lambda(v))/\lambda(v)$ to a change in δ:

$$\frac{\partial}{\partial\,\delta}\left(\frac{\delta + \lambda(v)}{\lambda(v)}\right) = \frac{\lambda(v) - \delta\,\lambda'(v)\,\partial\,v\,/\,\partial\,\delta}{\lambda(v)^2} > 0,$$

where $\partial\,v\,/\,\partial\,\delta < 0$ is derived from (21). These results indicate that the proportion of the wage offers greater than $\hat{w}$ rises when the separation rate of a match is sufficiently low (see Figure 2). [17] For such a low δ, it follows from (20) that the highest wage $\bar{w}$ goes up. When a job match does not separate very often, a firm with a vacant job obtains more expected profits (represented by the RHS of (21)) than before and has incentive to offer higher wages. One interpretation of the low separation rate is that the economy is in a boom and firms are not likely to become bankrupt in this situation. We conclude that a reduction in standard hours will be effective as a means of increasing high-paying jobs in a

16) It follows from (27)(28) and (29) that the latter part of (26) is decreasing in $\bar{l}$ for any $w > \hat{w}$.

17) Note that the wage offer distribution evaluated at a lower δ (depicted by $\Gamma_2(w)$) dominates the original distribution (depicted by $\Gamma_1(w)$) by the first degree: $\Gamma_2(w) \leq \Gamma_1(w)$ for any w.

boom, as well as in reducing unemployment. [18] By reducing $\bar{l}$, it follows from Proposition 2 that the number of vacancies increases under low β. This gives employers an incentive to pay higher wages because such wages enable them to find job seekers and prevent employed workers from moving to other jobs. Since it is difficult for firms with a vacant job to find trading partners when the vacancy rate is high, paying higher wages will be desirable.

6 Conclusions

We have developed a wage-posting model combined with a matching framework to examine the effects of a reduction in standard hours. In contrast to traditional works based on a matching model with an ex-post-bargaining mechanism, this paper assumes a unilateral wage determination by employers and the posting of wage offers before the opening of the search process. In this model, actual working time is decided by employers. Therefore, workers (both employed and unemployed) are not involved in determining the contents of a contract. Such situations correspond to a case in which workers do not have enough bargaining power because they are inexperienced, have few skills, or have highly specialized skills that are not applicable outside their current jobs. In this situation, a reduction in standard hours can decrease unemployment, provided that overtime work is not fully compensated and that unemployed workers are poorly compensated in the economy. Since unpaid overtime is a prevailing phenomenon in many developed countries, the results obtained in this paper are applicable to countries in which employers have strong control over wage determination and only a small part of overtime work is compensated. Accordingly, this paper demonstrates that a policy of reducing standard hours as one method of work sharing will, with some conditions, reduce unemployment.

In addition, we have shown that (i) unemployment can also be reduced by forcing firms to give workers more compensation for overtime work; (ii) the work time reduction policy may increase the number of jobs that post higher wages, especially when the separation rate of a match is low. It follows that it is possible to reduce unemployment and increase the number of high-paying jobs simultaneously by reducing standard hours. Therefore, we conclude that this reduction policy will be an effective way of performing work sharing.

Appendix

A. Computation of $\partial(w_R/\bar{l})/\partial\bar{l}$:

First, a change of $\bar{l}$ has an impact of w_R as follows:

$$\frac{\partial w_R}{\partial \bar{l}} = \frac{\gamma^2}{(1-\gamma)\,\bar{l}} \left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{\gamma/(1-\gamma)} \left[\frac{2}{\gamma^2}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)} - 1\right]$$
$$\Bigg/ \left\{1 - \beta + \frac{\beta\,\gamma}{(1-\gamma)\,\bar{l}} \left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)} \left[\frac{2}{\gamma^2}\left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)} - 1\right]\right\}. \quad \text{(A.1)}$$

18) Even if the density of every offer declines when the standard hours are reduced, the degree of such a decline for higher wage offers is smaller than that for lower offers. Thus, we can say that a proportion of wage offers greater than $\hat{w}$ is high when δ is low.

It follows from (A.1) that the elasticity of w_R with respect to $\bar{l}$ is less than 1:

$$\frac{\bar{l}}{w_R}\,\frac{\partial w_R}{\partial \bar{l}} - 1$$
$$= -\ (1-\beta) \Big/ \left\{ 1-\beta + \frac{\beta\,\gamma}{(1-\gamma)\,\bar{l}} \left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{1/(1-\gamma)} \left[\frac{4}{\gamma^2} \left(\frac{\bar{l}\,\gamma}{\beta\, w_R}\right)^{(2-\gamma)/(1-\gamma)} - 1 \right] \right\}$$
$$<\ 0, \tag{A.2}$$

provided that condition (11′) is satisfied. Note that (A.2) results in $\partial(w_R/\bar{l})/\partial\,\bar{l} < 0$ and $\partial l(w_R)/\partial\bar{l} > 0$. That is, reducing the standard hours also diminishes the total number of hours worked for the jobs that pay w_R. This is consistent with the aim of work sharing.

B. The derivation of (27):

$$\frac{\partial}{\partial \bar{l}} \left[\frac{(\beta/\bar{l})(\bar{l}\gamma/\beta\, w)^{1/(1-\gamma)} + (1-\beta)}{\sqrt{(1-\gamma)(\bar{l}\,\gamma/\beta\, w)^{\gamma/(1-\gamma)} - (1-\beta)\, w}} \right]$$
$$= \left\{ \frac{\beta\,\gamma}{1-\gamma}\, \bar{l}^{(2\gamma-1)/(1-\gamma)} \left(\frac{\gamma}{\beta\, w}\right)^{1/(1-\gamma)} \sqrt{(1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} - (1-\beta) w} \right.$$
$$- \frac{\gamma}{2}\, \bar{l}^{(2\gamma-1)/(1-\gamma)} \left(\frac{\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} \left[(1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w \right]^{-1/2}$$
$$\left. \times \left[\left(\frac{\beta}{\bar{l}}\right) \left(\frac{\bar{l}\gamma}{\beta\, w}\right)^{1/(1-\gamma)} + (1-\beta) \right] \right\} \Big/ \left[(1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w \right]. \tag{B.1}$$

Its numerator can be arranged as

$$\frac{\gamma}{1-\gamma}\, \bar{l}^{(2\gamma-1)/(1-\gamma)} \left(\frac{\gamma}{\beta\, w}\right)^{1/(1-\gamma)} \left[\frac{\beta\,(1-\gamma)}{2} \left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} - \frac{\beta\,(1-\beta)(1+\gamma)}{2\,\gamma}\, w \right].$$

We can then rewrite (B.1) as

$$(B.1) = \frac{\gamma}{1-\gamma}\, \bar{l}^{(2\gamma-1)/(1-\gamma)} \left(\frac{\gamma}{\beta}\right)^{1/(1-\gamma)} \left[\frac{\beta\,(1-\gamma)}{2} \left(\frac{\bar{l}\,\gamma}{\beta}\right)^{\gamma/(1-\gamma)} w^{(\gamma+1)/(\gamma-1)} \right.$$
$$\left. - \frac{\beta\,(1-\beta)(1+\gamma)}{2\,\gamma}\, w^{\gamma/(\gamma-1)} \right] \Big/ \left[(1-\gamma)\left(\frac{\bar{l}\,\gamma}{\beta\, w}\right)^{\gamma/(1-\gamma)} - (1-\beta)\, w \right]^{3/2}.$$

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Supplementary Materials

A. Derivation of (10)

$$
\begin{aligned}
x(w) \;&\equiv\; \gamma \left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{\gamma/(1-\gamma)} + (1-\beta)\,w - \left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{2/(1-\gamma)}, \\
\Rightarrow\;\; \frac{d\,x}{d\,w} \;&=\; -\,\frac{\beta\,\gamma}{\bar{l}\,(1-\gamma)} \left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{1/(1-\gamma)} + (1-\beta) \;+\; \frac{2\,\beta}{\bar{l}\,\gamma(1-\gamma)} \left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{(3-\gamma)/(1-\gamma)}, \\
&=\; 1-\beta + \frac{\beta}{\bar{l}\,(1-\gamma)} \left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{1/(1-\gamma)} \left[\frac{2}{\gamma}\left(\frac{\bar{l}\,\gamma}{\beta\,w}\right)^{(2-\gamma)/(1-\gamma)} - \gamma\right].
\end{aligned}
$$

B. Derivation of (16)

It follows from (14) and (15) that

$$
\begin{aligned}
u + (1-u)\,G(w) \;&=\; u + \frac{u\,\lambda(v)\,[\,\Gamma(w) - \Gamma(w_R)\,]}{\delta + \lambda(v)\,[\,1 - \Gamma(w) + \Gamma(w_R)\,]}, \\
&=\; u\left\{1 + \frac{\lambda(v)\,[\,\Gamma(w) - \Gamma(w_R)\,]}{\delta + \lambda(v)\,[\,1 - \Gamma(w) + \Gamma(w_R)\,]}\right\}, \\
&=\; u\left\{\frac{\delta + \lambda(v)}{\delta + \lambda(v)\,[\,1 - \Gamma(w) + \Gamma(w_R)\,]}\right\}.
\end{aligned}
$$

Substituting (13) into u in the above expression results in (16) (we use $\Gamma(w_R) = 0$).

C. Derivation of (19)

It follows from (18) that

$$
\begin{aligned}
&\frac{(1-\gamma)(\beta\,w/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w_R} \;=\; \frac{\{\delta + \lambda(v)\,[\,1 - \Gamma(w)\,]\}^2}{[\delta + \lambda(v)]^2}, \\
\Rightarrow\;\; &\sqrt{\frac{(1-\gamma)(\beta\,w/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w_R}} \;=\; \frac{\delta + \lambda(v)\,[\,1 - \Gamma(w)\,]}{\delta + \lambda(v)}, \\
\Rightarrow\;\; &[\delta + \lambda(v)\,]\sqrt{\frac{(1-\gamma)(\beta\,w/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w_R}} \;=\; \delta + \lambda(v)\,[\,1 - \Gamma(w)\,], \\
\Rightarrow\;\; &\lambda(v)\,\Gamma(w) \;=\; [\,\delta + \lambda(v)\,] \;-\; [\delta + \lambda(v)\,]\sqrt{\frac{(1-\gamma)(\beta\,w/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w_R}}, \\
\Rightarrow\;\; &\Gamma(w) \;=\; \left(\frac{\delta + \lambda(v)}{\lambda(v)}\right)\sqrt{\frac{(1-\gamma)(\beta\,w/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)} - (1-\beta)\,w_R}}.
\end{aligned}
$$

D. Derivation of (20)

Evaluating (19) at $w = \bar{w}$, we have

$$
\begin{aligned}
1 \;&=\; \frac{\delta+\lambda(v)}{\lambda(v)}\left[1-\sqrt{\frac{(1-\gamma)(\beta\,\bar{w}/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,\bar{w}}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,w_R}}\,\right],\\
\Rightarrow\;\frac{\lambda(v)}{\delta+\lambda(v)} \;&=\; 1-\sqrt{\frac{(1-\gamma)(\beta\,\bar{w}/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,\bar{w}}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,w_R}},\\
\Rightarrow\;\frac{\delta}{\delta+\lambda(v)} \;&=\; \sqrt{\frac{(1-\gamma)(\beta\,\bar{w}/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,\bar{w}}{(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,w_R}},\\
\Rightarrow\;&(1-\gamma)(\beta\,\bar{w}/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,\bar{w}\\
&=\left(\frac{\delta}{\delta+\lambda(v)}\right)^2\left[\,(1-\gamma)(\beta\,w_R/\bar{l}\,\gamma)^{\gamma/(\gamma-1)}-(1-\beta)\,w_R\,\right].
\end{aligned}
$$

E. The Condition for the Realizing Positive Employment Effects by Reducing $\bar{l}$

Differentiating the numerator of the RHS of (21) with respect to $\bar{l}$ yields

$$
\begin{aligned}
&-\frac{\gamma}{\bar{l}}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)\left(\frac{\bar{l}}{w_R}\frac{\partial w_R}{\partial \bar{l}}-1\right)-(1-\beta)\,\frac{\partial w_R}{\partial \bar{l}},\\
=\;&-\frac{\gamma}{\bar{l}}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\left(\frac{\bar{l}}{w_R}\frac{\partial w_R}{\partial \bar{l}}-1\right)-(1-\beta)\,\frac{\partial w_R}{\partial \bar{l}},
\end{aligned}
$$

It follows from (A.1) and (A.2) that

$$
\begin{aligned}
&\frac{\gamma}{\bar{l}}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\frac{1-\beta}{\Delta}-\frac{\gamma^2\,(1-\beta)}{\bar{l}(1-\gamma)\,\Delta}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\left[\frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}-1\right],\\
=\;&\frac{\gamma(1-\beta)}{\bar{l}\,\Delta}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\left\{1-\frac{\gamma}{1-\gamma}\left[\frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}-1\right]\right\},
\end{aligned}
$$

A sign of this expression depends on that of the term within the largest bracket in the second line. The positive employment effect is attained when

$$
1-\frac{\gamma}{1-\gamma}\left[\frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}-1\right]<0.
$$

In Subsection 5.2, we have found pairs of $(\bar{l},\,\beta)$ satisfying (24). The Area B in Figure 1 corresponds to pairs of these parameters satisfying the above inequality.

F. Derivation of (A.1)

Differentiating the LHS of (12) with respect to $\bar{l}$ results in

$$\frac{\gamma^2}{\gamma-1}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left(-\frac{\beta\,w_R}{\gamma\,\bar{l}^2}\right)-\frac{2}{\gamma-1}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}\left(-\frac{\beta\,w_R}{\gamma\,\bar{l}^2}\right),$$
$$=\frac{1}{1-\gamma}\left(\frac{\beta\,w_R}{\gamma\,\bar{l}^2}\right)\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left[\gamma^2-2\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}\right],$$
$$=\frac{\gamma^2}{\bar{l}(1-\gamma)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\left[1-\frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}\right].$$

By transposing this term, we obtain the numerator of (A.1).

G. Derivation of (24)

It follows from (23) that

$$\frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}-1=\frac{1-\gamma}{\gamma},$$
$$\Rightarrow\ \frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}=\frac{1}{\gamma},$$
$$\Rightarrow\ \left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}=\frac{\gamma}{2},$$
$$\Rightarrow\ \frac{\beta\,w_R}{\bar{l}\,\gamma}=\left(\frac{\gamma}{2}\right)^{-(1-\gamma)/(2-\gamma)},$$

H. Some Properties of the Reservation Wage

It follows from (12) that

$$\frac{\partial w_R}{\partial\beta}=-\left[\frac{\gamma\,w_R}{\bar{l}(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}-w_R-\frac{2\,w_R}{\bar{l}\,\gamma\,(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}\right]$$
$$\Big/\left[\frac{\beta\,\gamma}{\bar{l}(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}+(1-\beta)-\frac{2\,\beta}{\bar{l}\,\gamma\,(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}\right],$$

where

$$\frac{\partial\ \text{the LHS of (12)}}{\partial\,\beta}=\frac{\gamma\,w_R}{\bar{l}(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}-w_R-\frac{2\,w_R}{\bar{l}\,\gamma\,(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}.$$

These results lead to

$$\frac{\partial(\beta\,w_R)}{\partial\beta}=w_R+\beta\,\frac{\partial w_R}{\partial\beta}$$
$$=w_R\Big/\left[\frac{\beta\,\gamma}{\bar{l}(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}+(1-\beta)-\frac{2\,\beta}{\bar{l}\,\gamma\,(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}\right].$$

I. The Effect of the Change in β on the Vacancy Rate

Remember that the partial differentiation of the RHS of (20) with respect to β is equal in sign to

$$-\frac{1}{\bar{l}}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left(w_R+\beta\,\frac{\partial w_R}{\partial\beta}\right)-\frac{\partial w_R}{\partial\beta}+\left(w_R+\beta\,\frac{\partial w_R}{\partial\beta}\right). \qquad (*)$$

By using expressions for $\partial w_R/\partial\beta$ and $\partial(\beta\,w_R)/\partial\beta$, we can rewrite $(*)$ as follows:

$$\begin{aligned}
(*) \;=\; & -\frac{1}{\bar{l}}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\frac{w_R}{T} \\
& +\frac{1}{T}\left[\frac{\gamma\,w_R}{\gamma-1}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}-w_R-\frac{2\,w_R}{\bar{l}\,\gamma\,(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}\right] \\
& +\frac{w_R}{T}, \\
=\; & -\frac{1}{\bar{l}}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\frac{w_R}{T}+\frac{\gamma\,w_R}{\bar{l}\,T\,(1-\gamma)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left[\frac{2}{\gamma^2}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}-1\right], \\
=\; & \frac{w_R}{\bar{l}\,T\,(1-\gamma)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left[-(1-\gamma)-\gamma+\frac{2}{\gamma}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}\right], \\
=\; & \frac{w_R}{\bar{l}\,T\,(1-\gamma)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}\left[\frac{2}{\gamma}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(2-\gamma)/(\gamma-1)}-1\right],
\end{aligned}$$

where we denote T as

$$T \;\equiv\; \frac{\beta\,\gamma}{\gamma-1}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{1/(\gamma-1)}+(1-\beta)-\frac{2\,\beta}{\bar{l}\,\gamma\,(\gamma-1)}\left(\frac{\beta\,w_R}{\bar{l}\,\gamma}\right)^{(3-\gamma)/(\gamma-1)}.$$

J. Properties of the Wage Offer Density Function

(J-1) Differentiating $\Gamma(w)$ with respect to w results in

$$\begin{aligned}
\Gamma'(w) \;=\; & \frac{\delta+\lambda(v)}{2\,\lambda(v)}\left(\sqrt{\frac{(1-\gamma)\,\left(\beta\,w\,/\,\bar{l}\,\gamma\right)^{\gamma/(\gamma-1)}-w\,(1-\beta)}{(1-\gamma)\,\left(\beta\,w_R\,/\bar{l}\,\gamma\right)^{\gamma/(\gamma-1)}-w_R\,(1-\beta)}}\right)^{-1} \\
& \times\frac{(\beta/\bar{l})(\beta\,w/\bar{l}\,\gamma)^{1/(\gamma-1)}+(1-\beta)}{(1-\gamma)\,\left(\beta\,w_R\,/\bar{l}\,\gamma\right)^{\gamma/(\gamma-1)}-w_R\,(1-\beta)}.
\end{aligned}$$

Arranging this expression yields

$$\begin{aligned}
\Gamma'(w) \;=\; & \frac{\delta+\lambda(v)}{2\,\lambda(v)\,\sqrt{(1-\gamma)\,\left(\beta\,w_R\,/\bar{l}\,\gamma\right)^{\gamma/(\gamma-1)}-w_R\,(1-\beta)}} \\
& \times\left[\frac{(\beta/\bar{l})(\beta\,w/\bar{l}\,\gamma)^{1/(\gamma-1)}+(1-\beta)}{\sqrt{(1-\gamma)\,\left(\beta\,w\,/\,\bar{l}\,\gamma\right)^{\gamma/(\gamma-1)}-w\,(1-\beta)}}\right].
\end{aligned}$$

(J-2) The derivation of (29):

$$
\begin{aligned}
& \gamma\,(1-\gamma)\,\left(\frac{\beta}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\; w^{-1/(1-\gamma)} - (1-\beta)(1+\gamma) = 0, \\
\Rightarrow\;& \gamma\,(1-\gamma)\,\left(\frac{\beta}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)}\; w^{-1/(1-\gamma)} = (1-\beta)(1+\gamma), \\
\Rightarrow\;& \frac{\gamma\,(1-\gamma)}{(1-\beta)(1+\gamma)}\,\left(\frac{\beta}{\bar{l}\,\gamma}\right)^{\gamma/(\gamma-1)} \;=\; w^{1/(1-\gamma)}, \\
\Rightarrow\;& \hat{w} \;\equiv\; \left[\frac{\gamma\,(1-\gamma)}{(1-\beta)(1+\gamma)}\right]^{1-\gamma}\left(\frac{\beta}{\bar{l}\,\gamma}\right)^{-\gamma}.
\end{aligned}
$$